

Thynell, S. T., "Treatment of Radiation Heat Transfer in Absorbing, Emitting, Scattering Two-Dimensional Cylindrical Media," *Numerical Heat Transfer*, Vol. 17, 1990, pp. 449-472.

Fig. 1 Geometry of the two-dimensional planar scattering system.

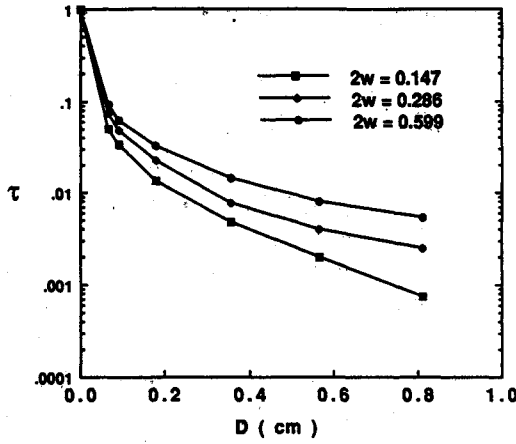


Fig. 2 Transmission data for LI900.

planar slab in this work. Extension to two-dimensional cylindrical geometry and anisotropically scattering materials will be presented in future publications.

II. Mathematical Model

Consider the zonal arrangement for a two-dimensional slab as shown in Fig. 1. In a transmission measurement, a line source of infinitesimal width dx_i and flux density I_0 is incident on a finite planar slab of width $2L$ and thickness D . A detector of width $2W$ (with $W < L$) is used to measure the transmitted radiation.

Based on the Hottel zonal method⁴ and standard network analysis technique,³ an energy balance at a volume V_i in Fig. 1 can be written as

$$4K \frac{V_i}{\omega} W_{gi} = ds_i g_i I_0 + \sum_{j=1}^N (g_i g_j) W_{gj} \quad (2)$$

with W_{gi} and W_{gj} being the net outgoing radiosity from volume V_i and V_j , respectively. $ds_i g_i$ is the source-to-volume exchange factor and $g_i g_j$ is the exchange factor from volume i to volume j . Since the objective of the analysis is to determine the effective transmission of the incident energy beam, all emissions (from the medium, the detector and the surrounding boundary) are neglected in the development of Eq. (2). Equation (2) can be readily inverted to yield

$$W_g = \left[4K \frac{\bar{V}}{\omega} - \bar{g} \bar{g} \right]^{-1} ds_i g_i I_0 \quad (3)$$

where W_g and $ds_i g_i$ are N -component radiosity and source-to-volume exchange factor vectors, respectively. $\bar{V}$ and $\bar{g} \bar{g}$ are the respective $N \times N$ volume matrix and volume-volume exchange factor matrix.

The net heat transfer Q_2 to the detector $2W$ in Fig. 1 is

$$Q_2 = I_0 ds_1 s_2 + \sum_{j=1}^N (s_2 g_j) W_{gj} \quad (4)$$

with $ds_1 s_2$ being the source-to-detector exchange factor, and $s_2 g_j$ being the volume-to-detector exchange factor. Combining Eqs. (3) and (4), the effective transmissivity of the medium becomes

$$\tau = \frac{1}{dx_1} [ds_1 s_2 + \omega (s_2 g)^T [4K \bar{V} - \omega \bar{g} \bar{g}]^{-1} (ds_i g)] \quad (5)$$

with $(s_2 g)^T$ being the transpose of the vector, $s_2 g$. In the above equations, various exchange factors are given by

$$s_2 g_i = \int_{V_i} \int_{2W} \frac{K dx_i dz_i dx_2 (D - z_i) S_2(Kt_{i2})}{t_{i2}^2} \quad (6)$$

$$ds_i g_i = dx_1 \int_{V_i} \frac{K dx_i dz_i z_i S_2(Kt_{i1})}{t_{i1}^2} \quad (7)$$

and

$$g_i g_j = \int_{V_i} \int_{V_j} \frac{K^2 dx_i dz_i dx_j dz_j S_1(Kt_{ij})}{t_{ij}} \quad (8)$$

with

$$t_{ij} = [(x_i - x_j)^2 + (z_i - z_j)^2]^{1/2} \quad (9)$$

and $S_n(Kt)$ defined in Ref. 5. For a collimated source, the exchange factor between the source and the detector, $ds_1 s_2$, is given by

$$ds_1 s_2 = dx_1 e^{-KD} \quad (10)$$

while for a diffuse source, the exchange factor becomes

$$ds_1 s_2 = dx_1 \int_{2W} \frac{dx_2 D^2 S_3(Kt_{21})}{t_{21}^3} \quad (11)$$

An important step in the evaluation of Eq. (5) is the numerical tabulation of the various exchange factors. It can be readily shown that with the established analytical and numerical properties of $S_n(Kt)$,⁵ many of the required numerical integrals can be simplified in many cases to single integrals, and evaluated with little effort. The detailed mathematical procedure is presented in an earlier publication.⁶

III. Experimental Procedure and Data Reduction

The experimental approach is straightforward. Specifically, transmission measurements for a collimated source at a specific wavelength (He-Ne Laser at 0.638 μm) are made with samples of different width, depth, and detectors of different window openings. The two-dimensional planar effect is simulated by shaping the incoming beam as a line source in the lengthwise direction of the sample. The detector opening is a rectangle with a finite width and a length identical to that of the sample. Standard least-square curve-fitting procedure is used to determine the best-fit values of the two unknown parameters, K and ω .

It is interesting to note that in the standard one-dimensional approach based on the Beer-Lambert law, measurements at moderate and large optical thicknesses are generally required to yield statistically reliable estimates of the extinction coefficient. Measurements for media with large optical thicknesses, however, are often unreliable because of the extremely small value of the transmissivity. The accuracy of the one-dimensional approach in determining K is thus naturally limited.

The experimental data presented in this work, on the other hand, show that the transmissivity can vary significantly with the detector window opening, even in samples with small optical depth. Since the measured value of the transmissivity is high, the relative error of measurement is small. The present approach should be a reliable technique in measuring optical properties, particularly for media with moderate or large scattering albedo.

Numerical results based on LI900 data are used as an example to demonstrate the unique combination of this experimental procedure and Eq. (5) to resolve optical properties. Two factors, however, limited the accuracy of the present calculation. First, LI900 is known to be a highly anisotropic forward-scattering material. Based on a Mie calculation for the known fiber size distribution of the test sample, for example, the backward-scattering fraction was estimated to be 0.173.⁷ The backward-scattering fraction for an isotropically scattering medium is expected to be 0.5. Second, the optical

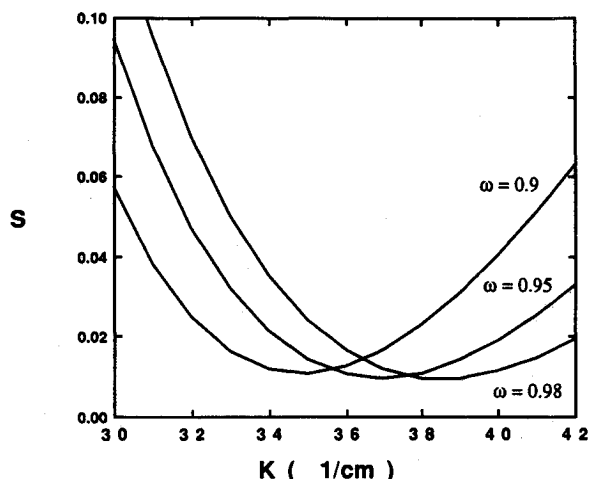


Fig. 3 Error sum calculated for different K and ω .

width of the scattering sample (KL) was not recorded in the experiment, therefore numerical data for a one-dimensional slab ($KL \rightarrow \infty$) are used in the calculation.

An initial estimate for the extinction coefficient K is obtained by interpolating the LI900 transmission data in the optically thin limit. For each assumed K and ω , an error sum is calculated by

$$S = \sum_{\text{data}} (\tau_{\text{measure}} - \tau_{\text{numerical}})^2 \quad (12)$$

with the summation extended to all measured data points. The behavior of S for different ω and K is shown in Fig. 3. It is apparent that the present approach is numerically stable. It yields a unique least-square prediction for $\omega = 0.95$ and $K = 37 \text{ cm}^{-1}$. A Mie calculation, based on the known fiber size distribution of the test sample, yields $\omega = 1.0$, $K = 148 \text{ cm}^{-1}$, and $b = 0.173$. The discrepancy in K can probably be attributed to the anisotropic scattering effect which is not included in the model and the uncertainty of the index of refraction utilized in the Mie calculation. Nevertheless, the agreement in ω and the ability of the model to predict the qualitative behavior of the transmissivity suggest that the current mathematical model and experimental approach is a promising procedure for the determination of optical properties. It is also apparent that this procedure can be readily extended to applications for anisotropically scattering materials.

IV. Conclusions and Future Works

A unique experimental and numerical procedure is shown to be effective in predicting optical properties for a two-dimensional absorbing and isotropically scattering medium. Analysis of some limited data generated for a fibrous insulation material (LI900) shows the potential of present approach.

Additional experiments are needed to verify the statistical reliability and sensitivity of the proposed experimental technique. Since most insulation materials scatter anisotropically, an extension of the numerical procedure to general absorbing, anisotropically scattering media is required. The recently developed generalized zonal method⁸ appears to be ideally suited for the extension. These efforts are currently underway and results will be reported in future publications.

References

- ¹Tong, T. W., and Tien, C. L., "Radiative Heat Transfer in Fibrous Insulations. Part I: Analytical Study," *ASME Journal of Heat Transfer*, Vol. 105, 1983, pp. 70-75.
- ²Cunnington, G. R., Tong, T. W., and Swathi, P. S., "Angular Scattering of Radiation from Cylindrical Fibers," *AIAA Paper 88-2721*, 1988.

³Yuen, W. W., "Development of a Network Analogy and Evaluation of Mean Beam Length for Multi-Dimensional Absorbing/Isotropically Scattering Media," *ASME Journal of Heat Transfer*, Vol. 112, May 1990, pp. 408-414.

⁴Hottel, H. C., and Sarofim, A. F., *Radiative Transfer*, McGraw-Hill, New York, 1967.

⁵Yuen, W. W., and Wong, L. W., "Numerical Computation of an Important Integral Function in Two-Dimensional Radiative Transfer," *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 29, No. 2, 1983, pp. 145-149.

⁶Yuen, W. W., Ma, A., Hsu, I. C., and Cunningham, G. R., "Evaluation of Optical Properties for Scattering/Absorbing Insulation Materials with Two-Dimensional Scattering," *AIAA Paper 90-1719*, June 18-20, 1990.

⁷Lockheed, private communication.

⁸Yuen, W. W., and Takara, E. E., "Development of a Generalized Zonal Method for the Analysis of Radiative Transfer in Absorbing and Anisotropically Scattering Media," the 1990 ASME/AIAA Thermophysics and Heat Transfer Conference, Seattle, WA, June 18-20, 1990.

Transient Laminar Forced Convection From a Circular Cylinder Using a Body-Fitted Coordinate System

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Introduction

THE problem of steady flow and heat transfer characteristics around a circular cylinder has been widely investigated. At the situation of Reynolds number $5 < Re < 40$ there exists a pair of steady and symmetric vortices right behind the cylinder. When $Re > 40$, the flow begins to have an unsteady motion. The vortices are oscillating periodically and flowing in the downstream direction. Thoman and Szweczyk,¹ Gresho et al.,² and Hwang et al.³ studied the time-dependent flow characteristics of a circular cylinder. Jordan and Fromm,⁴ Lin et al.,⁵ Patel,⁶ Lecoine and Piquet,⁷ and Smith and Brebbia⁸ have presented their numerical results for the unsteady incompressible viscous flow past a circular cylinder.

Jain and Goel,⁹ McAdam,¹⁰ Kramers,¹¹ Van der Hegge Zijnen,¹² and Tsubouchi and Masuda¹³ discussed the development of the vortex shedding behind a circular cylinder. Karniadakis¹⁴ used the spectral element method to solve forced convection heat transfer from an isolated cylinder in crossflow for Reynolds numbers up to 200. In the present paper, the problem of transient laminar forced convection from a horizontal isothermal cylinder is studied. The Navier-Stokes equations and the energy equation for an unsteady incompressible fluid flow are solved by using a body-fitted orthogonal coordinate system and spline alternating direction implicit (SADI) method, described by Rubin and Graves.¹⁵ The computed results are compared with previous experimental correlations and numerical results. Although there are many investigations concerning this subject, the main contribution of the present paper is that a different numerical technique is applied and the results are extended to $Re = 500$.

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